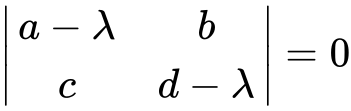
# 1.15 Eigenvalues and eigenvectors

**1a.** *[4 marks]*

## Markscheme

the eigenvalues satisfy

     ***M1***

      ***A1***

      ***A1***

the condition for real roots is

      ***M1***

      ***AG***

***[4 marks]***

**1b.** *[2 marks]*

## Markscheme

if the matrix is symmetric, *b* = *c*. In this case,       ***M1***



because each square term is non-negative      ***R1AG***

***[2 marks]***

**1c.** *[2 marks]*

## Markscheme

the characteristic equation is

     ***M1***

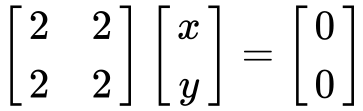
      ***A1***

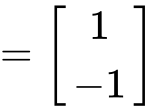
***[2 marks]***

**1d.** *[4 marks]*

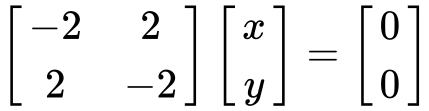
## Markscheme

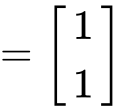
taking 

     ***M1***

giving eigenvector        ***A1***

taking 

     ***M1***

giving eigenvector        ***A1***

***[4 marks]***

**2a.** *[4 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(i)     ***M*** ***MM*** only exists if the number of columns of ***M*** equals the number of rows of ***M     R1***

hence ***M*** is square     ***AG***

(ii)     apply the determinant function to both sides     ***M1***

***M******M***

use the multiplicative property of the determinant

***M******M******M******M***     ***(M1)***

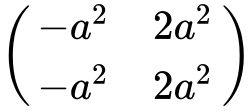
hence ***M*** or 1     ***A1***

***[4 marks]***

**2b.** *[12 marks]*

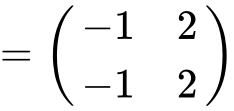
## Markscheme

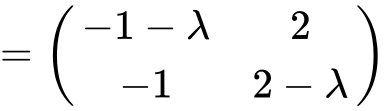
(i)     attempt to calculate ***N***        ***M1***

obtain         ***A1***

equating to ***N        M1***

to obtain         ***A1***

(ii)     ***N*** 

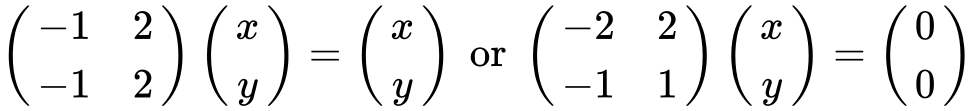
***N*** ***I***         ***M1***

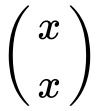
       ***(A1)***

       ***(A1)***

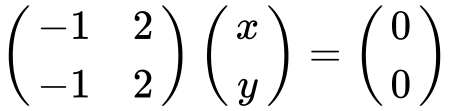
 is 1 or 0        ***A1***

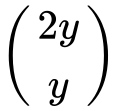
(iii)     let 

to obtain         ***M1***

hence eigenvector is         ***A1***

let 

to obtain         ***M1***

hence eigenvector is         ***A1***

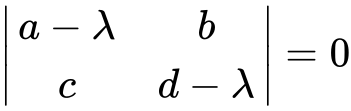
**Note:** Accept specific eigenvectors.

***[12 marks]***

**3.** *[12 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(a)     the eigenvalues satisfy      ***(M1)***

     ***A1***

using the sum and product properties of the roots of a quadratic equation     ***R1***

(***M***)     ***AG***

***[3 marks]***

(b)     let 

putting  and , consider     ***M1***

     ***A1***

therefore  is an eigenvalue     ***AG***

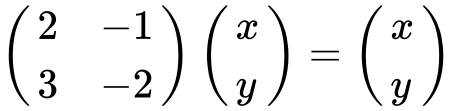
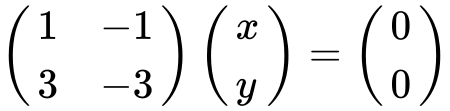
***[2 marks]***

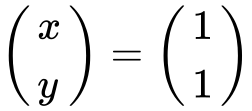
**Note:**Allow substitution for ,  into the quadratic equation for  followed by solution of this equation.

(c)     using any valid method     ***(M1)***

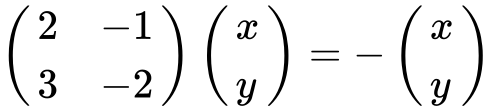
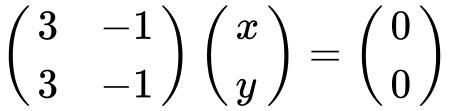
the eigenvalues are 1 and –1     ***A1***

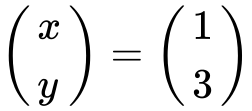
an eigenvector corresponding to  satisfies

 **or**      ***M1A1***

 **or** any multiple     ***A1***

an eigenvector corresponding to  satisfies

 **or**      ***M1***

 **or** any multiple     ***A1***

**Note:**Award ***M1A1A1*** for calculating the first eigenvector and ***M1A1*** for the second irrespective of the order in which they are calculated.

***[7 marks]***

Printed for SANSKAR SCHOOL

© International Baccalaureate Organization 2019

International Baccalaureate® - Baccalauréat International® - Bachillerato Internacional®